ECS455 Chapter 2

Cellular Systems

2.4 Traffice Handling Capacity and Erlang B Formula

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Capacity Concept: A Revisit

Trunking

- **Trunking**
- Allow a large number of users to **share** the relatively small number of channels in a cell by providing access to each user, **on demand**, from a **pool** of available channels.
- Exploit the **statistical behavior** of users
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms

For one user, A_u

amount of traffic

Traffic load

- ✓ **Traffic Intensity**: Measure of channel time utilization, which is the average channel occupancy measured in Erlangs.
  - This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels.
  - Denoted by *A*.
- **Holding Time**: Average duration of a typical call. Denoted by H = 1/μ.
- **Blocked Call**: Call which cannot be completed at time of request, due to congestion. Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the P_b probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.
- ✓ **Request Rate**: The average number of call requests per unit time. Denoted by λ.

λ_u

M/M/m/m Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process.***
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration** T **of the time that a user occupies a channel is exponentially distributed**, so that longer calls are less likely to occur.

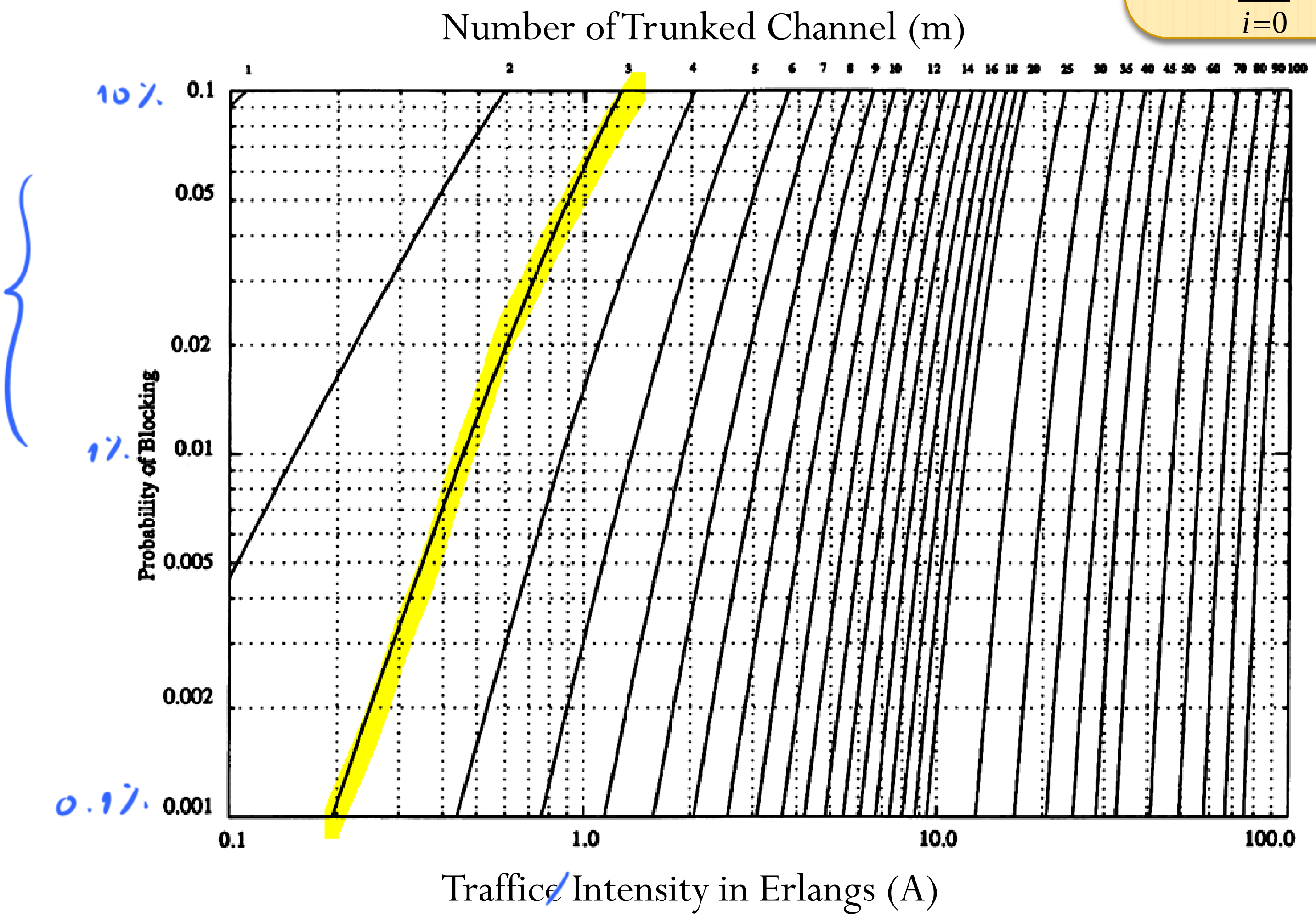
$f_T(t) = \mu e^{-\mu t}$

$t > 0$

- There are *m* channels available in the trunking pool.
  - For us, *m* = the number of channels for a cell (~~C~~) or for a sector

Erlang B Chart

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$



Example 1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

$m =$ (a) 5

$m =$ (b) 10

- Assume each user generates 0.1 Erlangs of traffic.

A_u

Ex. 6 times/day

average 24 min.

$$\frac{6}{\text{day}} \times 24 \text{ min} = \frac{6 \times \cancel{24}}{\cancel{24} \times 60} = \frac{1}{10}$$

$m = 5$

A	P_b
1.1	0.0045
$\boxed{1.13}$	0.0050

$\Downarrow$

11 users

$m = 10$

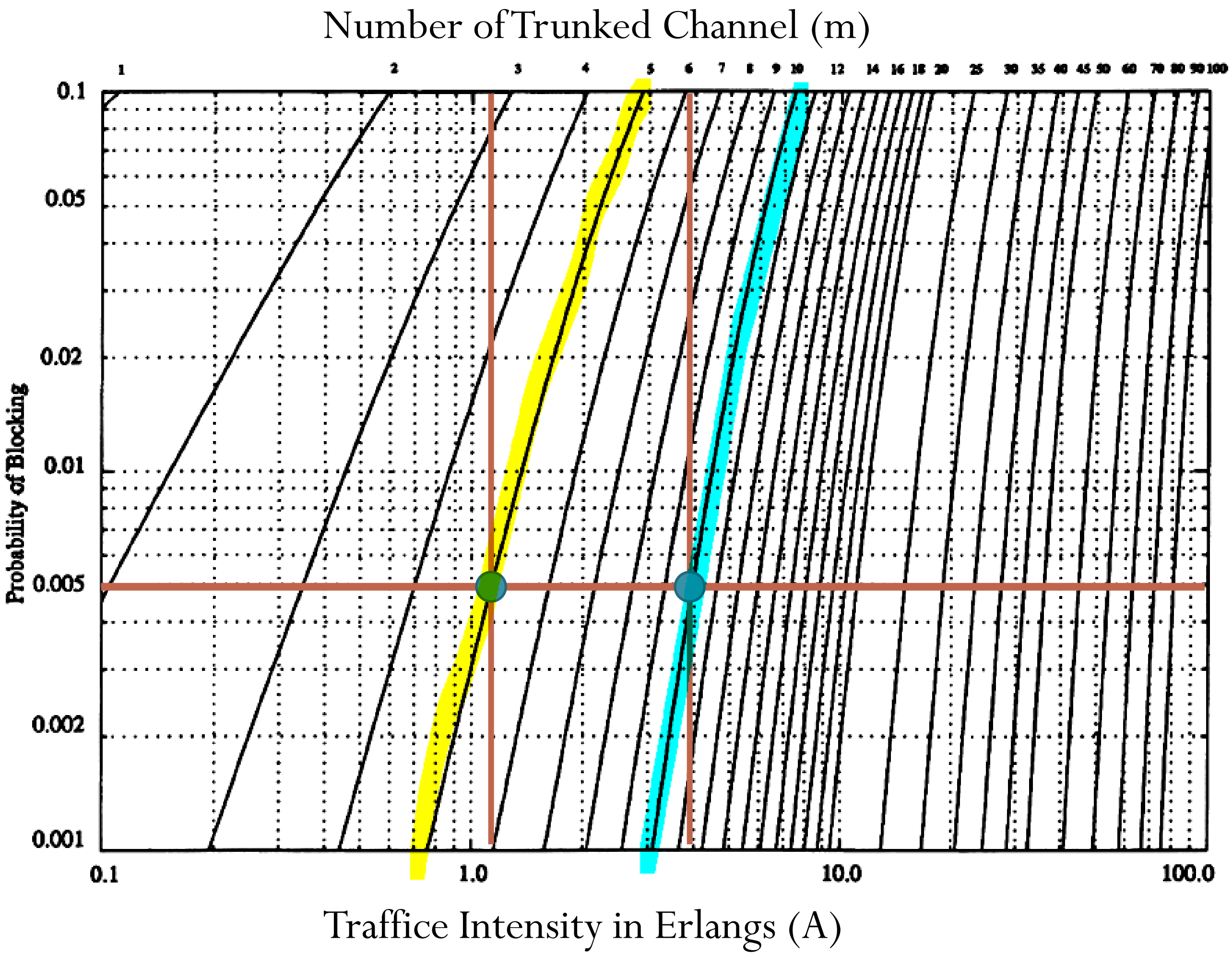
$\downarrow$

$A = 3.96$

$\Downarrow$

39 users

Erlang B


Number of Trunked Channel (m)
1
2
3
4
5
6
7
8
9
10
12
14
16
18
20
25
30
35
40
45
50
60
70
80
90
100
Probability of Blocking
0.1
0.05
0.02
0.01
0.005
0.002
0.001
0.1
1.0
10.0
100.0
Traffice Intensity in Erlangs (A)

Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes $\frac{1}{\mu} = 2 \text{ mins}$ $P_b = 0.01$
  - the probability of blocking is to be no more than 1%.

S ↘ $N = 7$

- If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell.
- From the Erlang B formula, can handle $\underline{\underline{44.2}}$ Erlangs or **1326 calls per hour**.

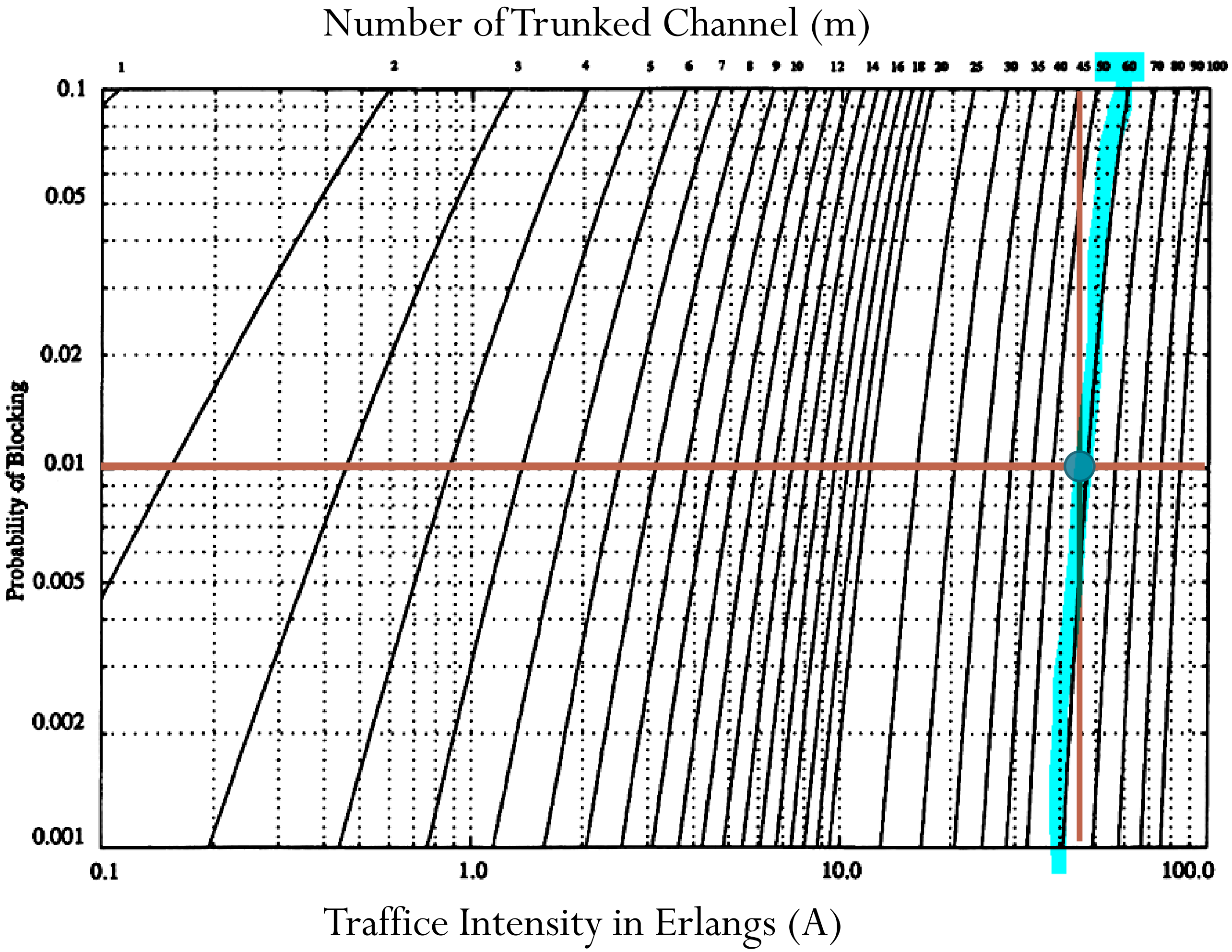
$$44.2 = \lambda \times \boxed{\frac{1}{\mu}}$$

$$= \lambda \times (2 \text{ mins/call})$$

$$\lambda = \frac{44.2 \text{ calls}}{2 \text{ min}} = 22.1 \frac{\text{calls}}{\text{min.}}$$

[Rappaport, 2002, Ex 3.9, p 92]

Erlang B


Number of Trunked Channel (m)
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2
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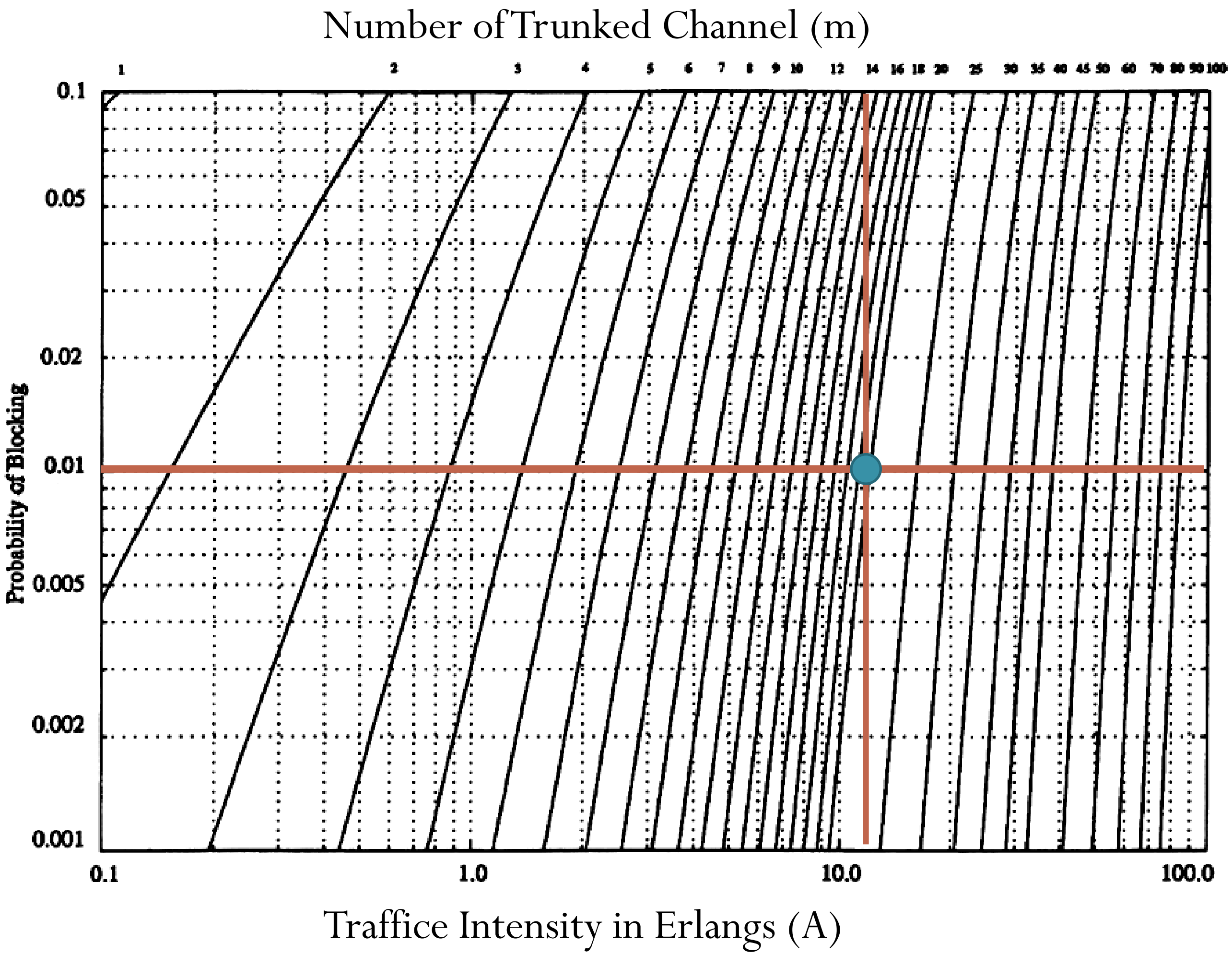
7
8
9
10
12
14
16
18
20
25
30
35
40
45
50
60
70
80
90
100
Probability of Blocking
0.1
0.05
0.02
0.01
0.005
0.002
0.001
0.1
1.0
10.0
100.0
Traffice Intensity in Erlangs (A)

Example 2.2

- Now employing **120° sectoring**, there are only 19 channels per antenna sector (57/3 # channels sector ~~antenna~~s).
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of 3 × 336 = **1008 calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the S/I for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]

Erlang B


Number of Trunked Channel (m)
1
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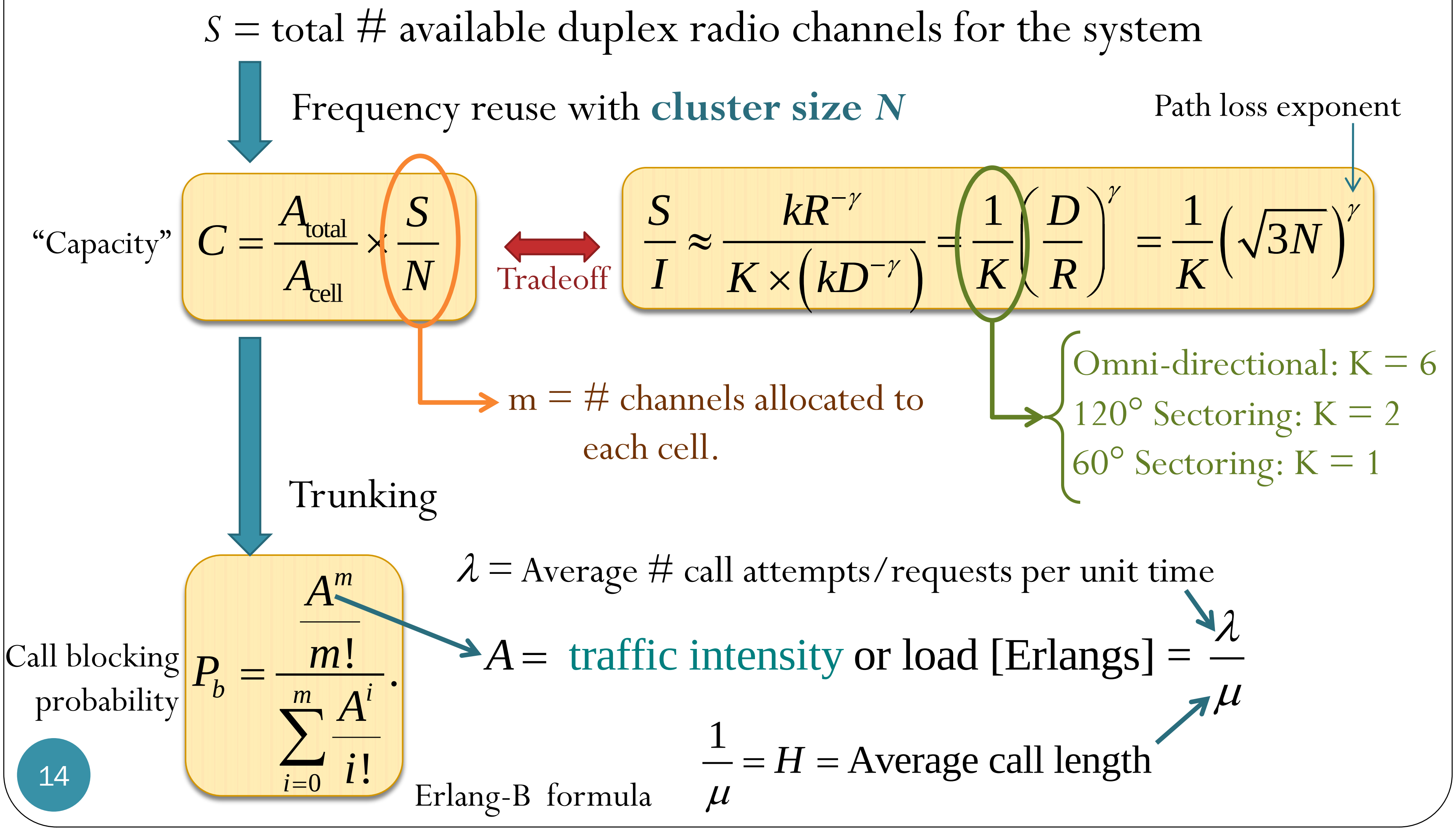
100.0
Traffice Intensity in Erlangs (A)

Erlang B Trunking Efficiency

Table 3.4 Capacity of an Erlang B System

Number of Channels ~~C~~ m	A Capacity (Erlangs) for GOS			
	= 0.01 1%	= 0.005	= 0.002	= 0.001 P_b
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	A = 4.46	3.96	3.43	3.09
20	A = 12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

Summary of Chapter 2: Big Picture



S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

Path loss exponent

"Capacity" $C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$

Tradeoff

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

Omni-directional: K = 6

120° Sectoring: K = 2

60° Sectoring: K = 1

Trunking

Call blocking probability

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Erlang-B formula

λ = Average # call attempts/requests per unit time

$A =$ traffic intensity or load [Erlangs] $= \frac{\lambda}{\mu}$

$\frac{1}{\mu} = H =$ Average call length

Example 3 (1)

- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20\times 10^6}{2\times 25\times 10^3} = 400 \text{ channels} \qquad \text{(FDD)}$$

- $\gamma = 4$
- Design requirements:
  - SIR $\geq$ 15 dB
  - $P_b \leq 5\%$

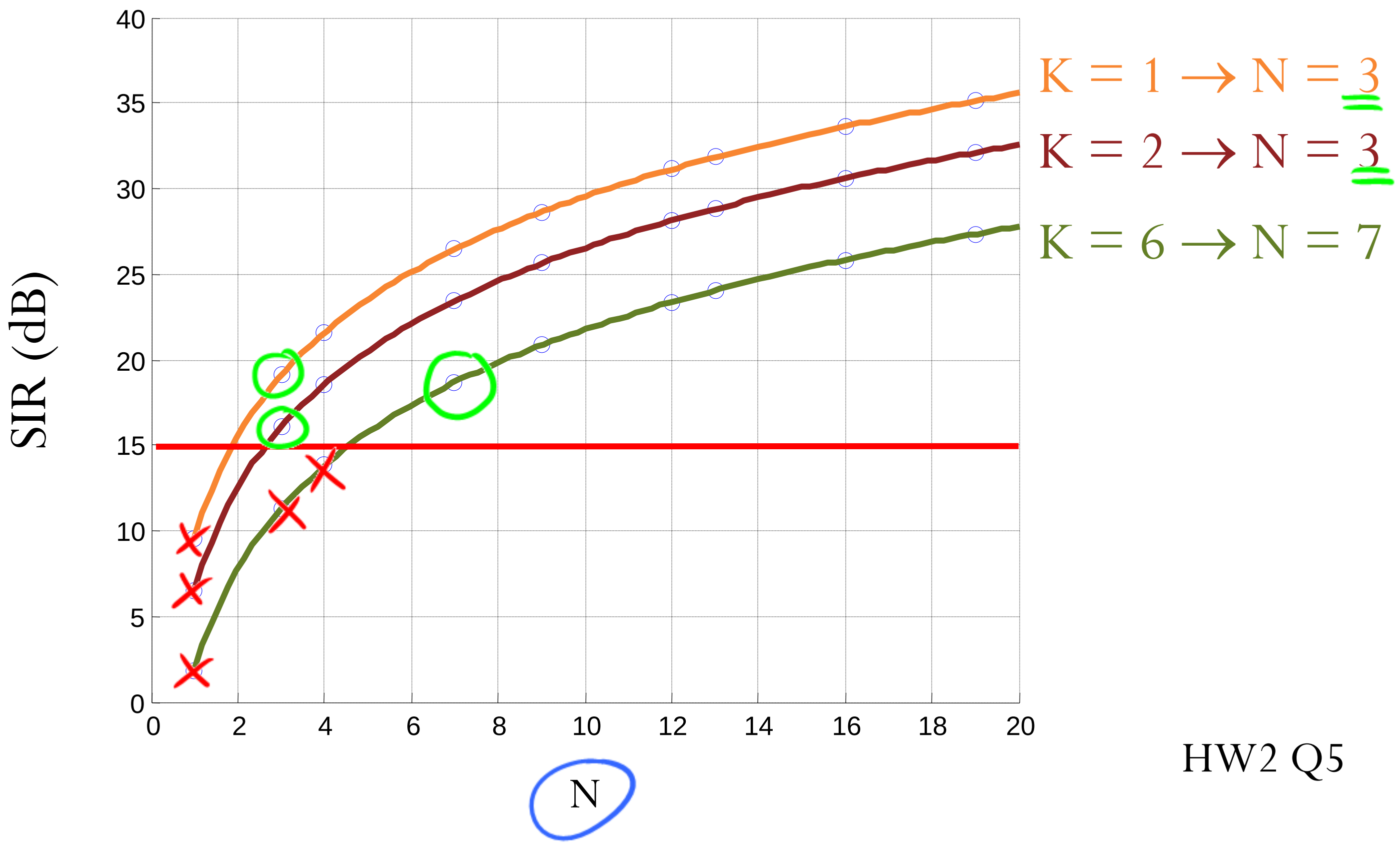
HW2 Q3

Example 3 (2)

- SIR ≥ 15 dB

$$\frac{S}{I} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
    N = [1, 3, 4, 7, 9, 12, 13, 16, 19];
    SIR = 10*log10(1/K*((sqrt(3*N)).^y))
    plot(N,SIR,'o')
    N = linspace(1,20,100);
    SIR = 10*log10(1/K*((sqrt(3*N)).^y));
    plot(N,SIR)
end
```



HW2 Q5

Example 3 (3)

15 dB → 120°
19 dB
11 dB

$P_b = 0.05$ →

	Om$_{\wedge}^{n}$idirectional	Sectoring (120°)	Sectoring (60°)
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
#channels/cell	400/7 = 57	400/3 = 133	400/3 = 133
#sectors	1	3	6
#channels/sector	57 = m	133/3 = 44	133/6 = 22
A [Erlangs]/sector	51.55	38.56	17.13
A [Erlangs]/cell	51.55	38.56×3 = 115.68	17.13×6 = 102.78
#users/cell	18558	41645	37001

Assume that each user makes 2 calls/day and 2 min/call on average → 1/360 Erlangs.

Make sure that you understand where numbers in the table come from!